Similar calculations must be done to improve the predictions of the "ρ-exchange model."

ACKNOWLEDGMENTS

The interest and advice of Professor O. Piccioni were invaluable for the success of this experiment. We wish to thank Professor W. Frazer, Dr. J. Fulco, Dr. W. Ramsay, and Professor D. Wong for many stimulating discussions. Thanks are due to Dr. Ralph Shutt and his group for the use of the 20-in. bubble chamber, to the Yale and Brookhaven groups for setting up the beam, to Dr. M. H. Blewett, Dr. H. Brown, Dr. R. Good, Dr. W. Mehlhop, and the 20-in. chamber crew for their help during the runs. The help of P. Yager, T. Hendricks, R. Mitra, and of our scanners and technicians is appreciated. The Western Data Processing Center has given us graciously many hours of IBM-7094 computer time. One of us (Duong-Nhu Hoa) would like to thank Professor F. Perrin for a travel scholarship.

PHYSICAL REVIEW VOLUME 136, NUMBER 1B 12 OCTOBER 1964

Relation Between Masses of Pseudoscalar Octet and Vector Octet*

JOSÉ R. FULCO AND DAVID Y. WONG†
University of California, San Diego, La Jolla, California

(Received 25 May 1964)

The effect of the splitting of a pseudoscalar octet on the splitting of a vector octet is investigated using a simple effective-range formula for the coupling of a vector to two pseudoscalars. Taking the observed masses of the pseudoscalar octet to be $\eta(548)$, $K(496)$, and $\pi(140)$, it is found that the mass differences among the members of the vector octet φ, K^*, and ρ give the order $\varphi > K^* > \rho$. The magnitude of the calculated splitting is approximately twice the observed values. It is also shown that to first order in the mass splitting, if the pseudoscalar octet satisfies the Gell-Mann–Okubo mass formula, then the vector octet also satisfies the GMO formula. Furthermore, a deviation of the pseudoscalar masses from the GMO formula implies a somewhat larger deviation for the vector masses with an opposite sign. This result is in qualitative agreement with experimental values.

It is well known that the octet of vector mesons ρ, K^*, and φ and the octet of pseudoscalar mesons π, K, and η do not satisfy the Gell-Mann–Okubo[1] mass formula exactly. In particular, the discrepancy for the vector mesons is larger than that for the pseudoscalar mesons. It is of some interest then to investigate the effect of the actual mass splitting of the pseudoscalar octet on the mass splitting of the vector octet[2] and, in particular, the consequences of the deviation of the pseudoscalar mesons from the GMO mass formula. Using the relativistic effective-range approximation for the coupling of a vector meson to two pseudoscalars, we show that taking the observed masses of the pseudoscalar octet to be $\eta(548)$, $K(496)$, and $\pi(140)$ the masses of the members of the vector octet are in the order $\varphi > K^* > \rho$. The magnitude of the calculated splitting is approximately twice the observed values. It is shown that to first order in the mass splitting, if the pseudoscalar octet satisfies the Gell-Mann–Okubo mass formula, then the vector octet also satisfies it. This is also true when the potentials are determined by the bootstrap mechanism. Furthermore, a deviation of the pseudoscalar masses from the GMO formula implies a larger deviation for the vector masses with the opposite sign. In other words, the $\eta(548)$ being lighter than the GMO prediction of 565 MeV implies that the φ should be heavier than the GMO prediction of 930 MeV. This result is in qualitative agreement with experimental values. However, as we shall see later, the violation of the GMO formula by the vector octet is substantially greater than the prediction derived from the first-order formula using observed values of η, K, and π.

Within the approximation of keeping only first-order terms in the mass differences, we examine several modifications of the effective-range formula. We find that these modifications do not change the result of the simple effective-range formula by more than 20 to 30%. On the other hand, in view of the large mass splitting within the pseudoscalar octet and the large value of the predicted first-order vector mass splitting, it appears that higher order terms in the mass differences could be quite important.

In the language of dispersion relations, the effective-range formula is a representation of the T matrix with the "potential" given by a simple pole in the unphysical region. In general, both the T matrix and the potential term are matrices of dimension equal to the number of channels having the same quantum numbers. For the

* Work supported in part by the U. S. Atomic Energy Commission.

† Alfred P. Sloan Fellow.

[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).

[2] The effect of the pseudoscalar mass differences has been considered by R. H. Capps, Phys. Rev. **132**, 2749 (1963); and Northwestern University, January 1964 (unpublished), assuming the bootstrap mechanism and using several modifications of the determinantal method.

present consideration, we have $\varphi \to K\bar{K}$, $K^* \to (K\pi, K\eta)$ and $\rho \to (\pi\pi, K\bar{K})$. As suggested by the qualitative success of the bootstrap model,[3] we consider potentials transforming like an 8 representation:

$$B^{(\varphi)} = [\Gamma/(s+s_0)], \tag{1}$$

$$B^{(K^*)} = [\Gamma/(s+s_0)]\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}, \tag{2}$$

$$B^{(\rho)} = [\Gamma/(s+s_0)]\begin{bmatrix} \frac{2}{3} & \frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} & \frac{1}{3} \end{bmatrix}, \tag{3}$$

where s is the center-of-mass energy squared, and Γ and s_0 are two parameters which determine the strength and the range of the interaction. With the explicit form of the potential given, one can solve for the T matrix using the ND^{-1} method. In the case of a simple pole for the potential, the solution takes the form

$$T = BD^{-1}, \tag{4}$$

$$D_{ij} = \delta_{ij} - \Gamma\beta_{ij} F(s, m_1^{(i)}, m_2^{(i)}), \tag{5}$$

where β_{ij} are constant matrix elements given by (1), (2), and (3),

$$F(s, m_1^{(i)}, m_2^{(i)}) = \frac{(s+s_0)}{\pi}\int_{(m_1^{(i)}+m_2^{(i)})^2}^{\infty} ds' \left(\frac{2q_i'^3}{s'^{1/2}}\right)\frac{1}{(s'+s_0)^2(s'-s)}, \tag{6}$$

q_i is the center-of-mass momentum for the ith channel coupled to pseudoscalars with masses $m_1^{(i)}$ and $m_2^{(i)}$. In the representation above, vector meson masses squared are just zeroes of the determinant of the D matrix.

Let us define the average pseudoscalar mass by

$$\bar{m}^2 = (m_\eta^2 + 4m_K^2 + 3m_\pi^2)/8 = 0.168\ (\text{BeV})^2. \tag{7}$$

Our results will be expanded in powers of mass differences

$$\delta\eta^2 = m_\eta^2 - \bar{m}^2 = 0.133\ (\text{BeV})^2, \tag{8}$$

$$\delta K^2 = m_K^2 - \bar{m}^2 = 0.078\ (\text{BeV})^2, \tag{9}$$

$$\delta\pi^2 = m_\pi^2 - \bar{m}^2 = -0.148\ (\text{BeV})^2. \tag{10}$$

Now for any given s_0, Γ is chosen so that in the limit of all pseudoscalar masses equal to $\bar{m}$, the determinants of the D matrices have a common zero at s equal to the empirical average vector mass squared:

$$\bar{m}_v^2 = (m_\varphi^2 + 4m_{K^*}^2 + 3m_\rho^2)/8 = 0.735\ (\text{BeV})^2. \tag{11}$$

Keeping this in mind, we find that in the neighborhood of $s = \bar{m}_v^2$

$$D^{(\varphi)} = -\Gamma(s - \bar{m}_v^2)F_s - 2\Gamma(\delta K^2)F_m, \tag{12}$$

$$|D^{(K^*)}| = -\Gamma(s - \bar{m}_v^2)F_s - \Gamma(\delta K^2 + \tfrac{1}{2}\delta\pi^2 + \tfrac{1}{2}\delta\eta^2)F_m, \tag{13}$$

$$|D^{(\rho)}| = -\Gamma(s - \bar{m}_v^2)F_s - \Gamma(\tfrac{4}{3}\delta\pi^2 + \tfrac{2}{3}\delta K^2)F_m, \tag{14}$$

where F_s is the partial derivative of F with respect to s evaluated at the point $(\bar{m}_v^2, \bar{m}^2, \bar{m}^2)$ and F_m is the partial derivative with respect to m_1^2 (or m_2^2) evaluated at the same point. The masses of the vector mesons are now given by

$$(m_\varphi^2 - \bar{m}_v^2) = 2(\delta K^2)R, \tag{15}$$

$$(m_{K^*}^2 - \bar{m}_v^2) = (\delta K^2 + \tfrac{1}{2}\delta\pi^2 + \tfrac{1}{2}\delta\eta^2)R, \tag{16}$$

$$(m_\rho^2 - \bar{m}_v^2) = (\tfrac{4}{3}\delta\pi^2 + \tfrac{2}{3}\delta K^2)R, \tag{17}$$

$$R = -(F_m/F_s).$$

Note that F_m and F_s are functions of s_0 only. By inspection of Eq. (6), one finds that F_m is negative while F_s is positive, thus R is positive definite. Furthermore, numerical evaluation of the integrals shows that R is an extremely insensitive function of s_0. In fact, R only varies from 2 to 3 monotonically for s_0 varying from 1 (BeV)^2 to 10^4 (BeV)^2.

Instead of taking s_0 as an arbitrary parameter, one can choose s_0 to give a V-PS-PS coupling constant equal to the average of the coupling constants deduced from the width of φ, K^*, and ρ. After factoring out the phase space factor, the coupling constants deduced from the widths are all approximately equal to 2 (this empirical fact can also be considered as a support for grouping φ, K^*, and ρ in an octet). The value of R evaluated this way is $R = 2.5$. Results of the vector mass differences are shown in Table I together with experimental values. It is seen that the sign and ordering of all the terms are in agreement with the data. The magnitudes of the calculated values are, however, substantially larger.

By taking appropriate combination of (15), (16), and (17) one arrives at the first-order mass formula

$$(4m_{K^*}^2 - 3m_\varphi^2 - m_\rho^2) = -\tfrac{2}{3}R(4m_K^2 - 3m_\eta^2 - m_\pi^2). \tag{18}$$

Since the GMO formula for the pseudoscalar octet requires the vanishing of the right-hand side of (18), the pseudoscalar mass formula implies a similar mass formula for the vector octet as long as one only keeps first-order terms in the mass differences of the pseudoscalar mesons. As we mentioned before, if m_η is smaller than the GMO prediction, we obtain m_φ larger than the value given by the mass formula.

If one takes Eq. (18) literally and inserts observed values of all the mesons on the left as well as on the right, then one finds that the ratio should be -8.5

TABLE I. Values of vector mass differences.

	Calculated values	Experimental values
$m_\varphi^2 - \bar{m}_v^2$	0.405 (BeV)^2	0.305 (BeV)^2
$m_{K^*}^2 - \bar{m}_v^2$	0.183 (BeV)^2	0.053 (BeV)^2
$m_\rho^2 - \bar{m}_v^2$	-0.325 (BeV)^2	-0.173 (BeV)^2

[3] R. H. Capps, Phys. Rev. Letters **10**, 312 (1963).

rather than our value of -1.7. However, if one includes higher order terms on the right, the required value of R could change greatly since the value of $(4m_K^2-3m_\eta^2-m_\pi^2)$ is very small and the higher order terms in general do not contain this factor.

We now turn to the question of what improvements over the simple effective-range formula can one make while keeping only first order terms in the mass difference. The following possibilities are considered, one at a time.

(1) *Potentials transforming like representations of* SU_3 *other than the* 8.—It can be shown that the mass formula is determined by the eigenvectors of the potential matrices rather than the individual matrix elements. These eigenvectors are common to all potentials transforming like any representations of SU_3. The only effect of adding other potentials is that the energy dependence will be changed and the definition of R have to be modified to certain weighed average of various (F_m/F_s). These, as we have seen, are very insensitive functions of the energy s. Thus, the effect will be small.

(2) *Fixed position of the interaction pole in the* q^2 *plane.*—For a potential of a given range, the position of the left-hand singularities are fixed in the q^2-plane rather than the s-plane if we allow the external mass to vary. Using the q^2 variable makes the calculation of the two-channel T-matrix more complicated. We have performed this calculation and find that the effect on R is only of a few percent.

(3) *Higher mass channels.*—Having determined the position and residue of the "potential" pole to fit the average mass and coupling constant of the vector mesons, one may ask whether the exchange of the vector meson in the crossed channel will give a potential comparable to the one we used. In other words, are the vector mesons produced primarily by a bootstrap mechanism? The answer is that the vector meson exchange gives rise to a potential substantially weaker than the phenomenological pole. This can be interpreted as an indication of the importance of higher mass states in the cross channel and the direct channel. The effect of the latter can be investigated by including a phenomenological high-mass channel in the formulation of the T matrix. For similicity we use a pole at the same position $s=-s_0$ for the potential in this additional channel and fix the rest mass of each particle at the value of the average baryon mass (1.2 BeV). Furthermore, wechoose the off-diagonal element of the potential matrix to be such that only the 8 representation is nonvanishing. For the potential in the original PS-PS channels, we normalize the residue of the pole to fit the vector exchange potential with $(g_v^2/4\pi)=2$. The strength of the higher mass potential and s_0 are now adjusted to fit the position and coupling constant of the "average" vector meson. The result gives a value of R approximately 20% higher than our previous value.

(4) *Potentials determined by bootstrap.*—We now consider the fact that because of the splitting of the vector octet and the pseudoscalar octet the potentials do not necessarily transform like a representation of SU_3. The bootstrap mechanism is a convenient formalism for the study of such effects. We first write down the bootstrap equations with an invariant potential plus small undetermined correction. Then we calculate this small potential by requiring self consistency in the bootstrap equation. The result shows an increment of R by approximately 30%. We note that the first order GMO mass formula is still satisfied with the modification of the potential by the bootstrap.[4]

In addition to the above considerations, we also find that evaluating all the integrals without expanding in powers of the pseudoscalar mass differences only changes the vector meson mass differences by approximately 20 to 30%. Our final conclusion is that the first-order mass splitting of the vector octet, as derived from dispersion relations, is in qualitative agreement with the observed φ, K^*, and ρ. Higher order contributions together with the modifications discussed above should be examined in detail before one can conclude whether the observed masses of φ, K^*, and ρ are compatible with the masses of η, K, and π. Preliminary calculation of higher order terms taking into account bootstrap contributions indicates that these terms are rather sizeable. If one can explain the large deviation of φ, K^*, and ρ from the GMO formula in terms of higher order corrections, then the ω-φ mixing hypothesis would no longer be needed.

[4] The result obtained by Capps (Ref. 1) is not consistent with the GMO formula.